EECE 310 - Lecture 15

Outline

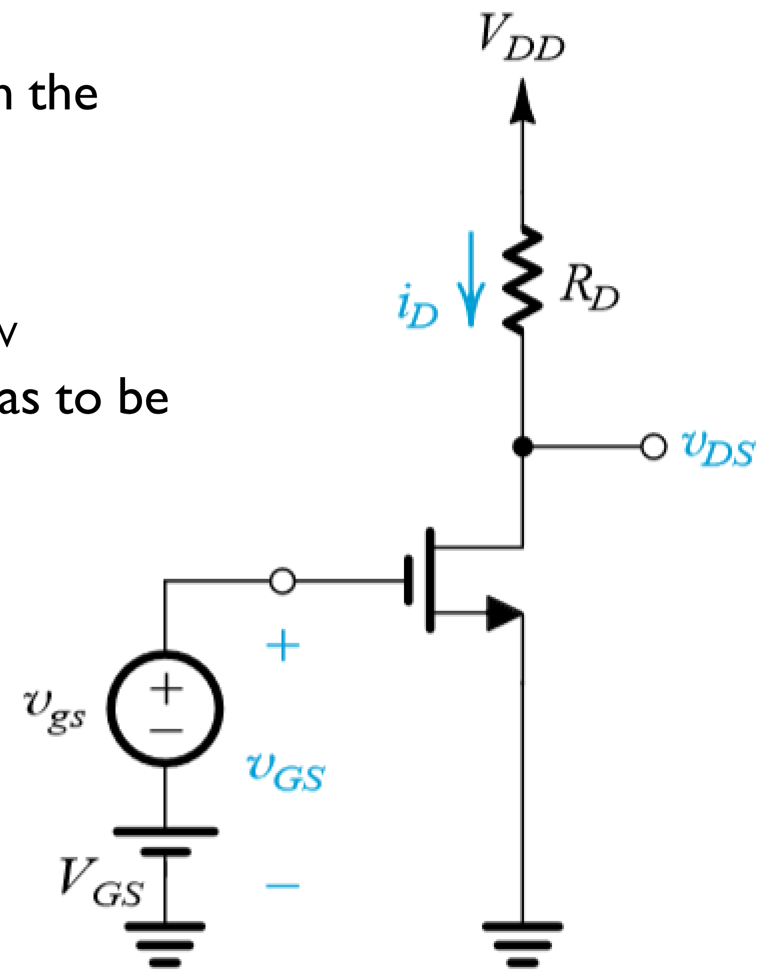
- **Small-Signal Operation and Models**

Small-Signal Operation and Models

- linear amplification can be obtained by biasing the MOSFET to operate in the saturation region and by keeping the input signal small
- we explore the small-signal operation
- MOS transistor is biased by applying a dc voltage V_{GS}
- input signal to be amplified, v_{gs}, is superimposed on the dc bias voltage V_{GS}
- output voltage is taken at the drain

The DC Bias Point

- $I_D = ½ * k_n(V_{GS}-V_t)^2 = ½ * k_n V^2_{OV}$
- The overdrive voltage $V_{OV} = V_{GS} - V_t$ at which the MOSFET is biased to operate
- DC Voltage at drain $V_{DS} = V_{DD} – R_D I_D$
- For saturation-region operation → $V_{DS} > V_{OV}$
- To allow for the required signal swing V_{DS} has to be sufficiently greater than V_{OV}



The Signal Current in the Drain Terminal

- Total instantaneous gate-to-source voltage will be: $v_{GS} = V_{GS} + v_{gs}$
- Total instantaneous drain current i_D,

$$i_D = \tfrac{1}{2} k_n (V_{GS} + v_{gs} - V_t)^2$$
$$= \underbrace{\tfrac{1}{2} k_n (V_{GS} - V_t)^2}_{I_D} + \underbrace{k_n (V_{GS} - V_t) v_{gs}}_{i_d} + \tfrac{1}{2} k_n v^2_{gs}$$

- where
  - I_D is the DC bias current
  - i_d represents a current component that is directly proportional to the input signal v_{gs}
  - The third term is a current component that is proportional to the square of the input signal. This component is undesirable because it represents nonlinear distortion
    → to reduce this distortion, the input signal should be kept small i.e. small signal

The Signal Current in the Drain Terminal

- So we need:

  $\frac{1}{2} k_n v^2_{gs} << k_n (V_{GS} - V_t) v_{gs}$

  $\rightarrow v_{gs} << 2 (V_{GS} - V_t) \rightarrow v_{gs} << 2V_{OV}$
- If this small-signal condition is satisfied, we may neglect the last term

  $\rightarrow i_D \approx I_D + i_d$ where $i_d = k_n (V_{GS} - V_t) v_{gs}$
- MOSFET transconductance:

  $g_m \equiv i_d / v_{gs} = k_n (V_{GS} - V_t) \rightarrow g_m = k_n V_{OV}$
- It can be shown that $g_m = 2I_D/V_{OV}$ (even with channel length modulation)
- g_m is equal to the slope of the $i_D - v_{GS}$ characteristic at the bias point,

$$g_m \equiv \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS} = V_{GS}}$$

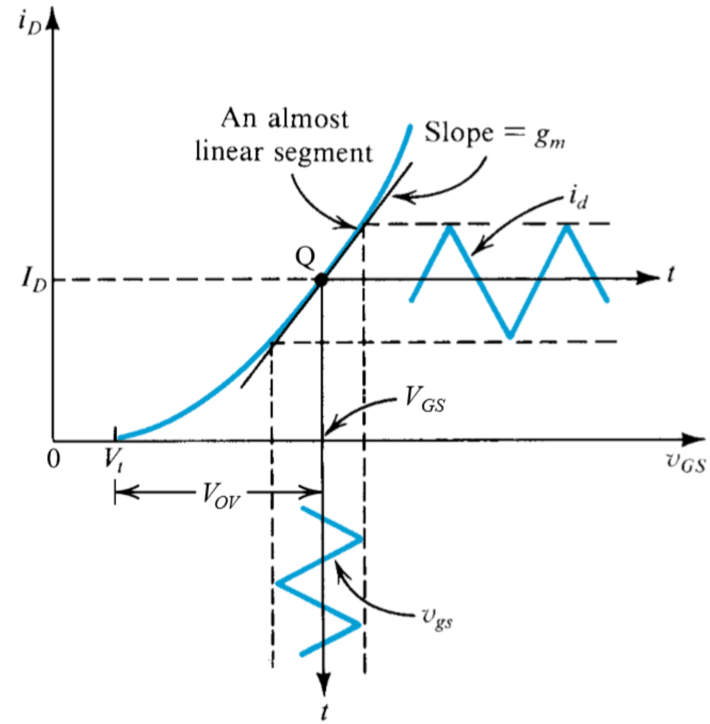


Figure 5.35 Small-signal operation of the MOSFET amplifier.

The Voltage Gain

- $v_{DS} = V_{DD} - R_D i_D$
- Under small signal condition:

$$v_{DS} = V_{DD} - R_D(I_D + i_d) \quad \rightarrow v_{DS} = V_{DS} - R_D i_d$$

$$\rightarrow v_{ds} = -i_D R_D = -g_m v_{gs} R_D$$

$$\rightarrow A_v \equiv v_{ds} / v_{gs} = -g_m R_D$$

The minus sign indicates that the output signal v_{ds} is 180° out of phase with respect to the input signal v_{gs}

The Voltage Gain

- The input signal is assumed to have a triangular waveform
- amplitude << $2(V_{GS} - V_t)$, the small-signal condition (for linear operation)
- For saturation region at all times →
  a. minimum value of v_{DS} should not fall below the corresponding value of v_{GS} by more than V_t
  b. maximum value of v_{DS} should be smaller than V_{DD}; (otherwise cutoff region and the peaks of the output signal waveform will be clipped off)

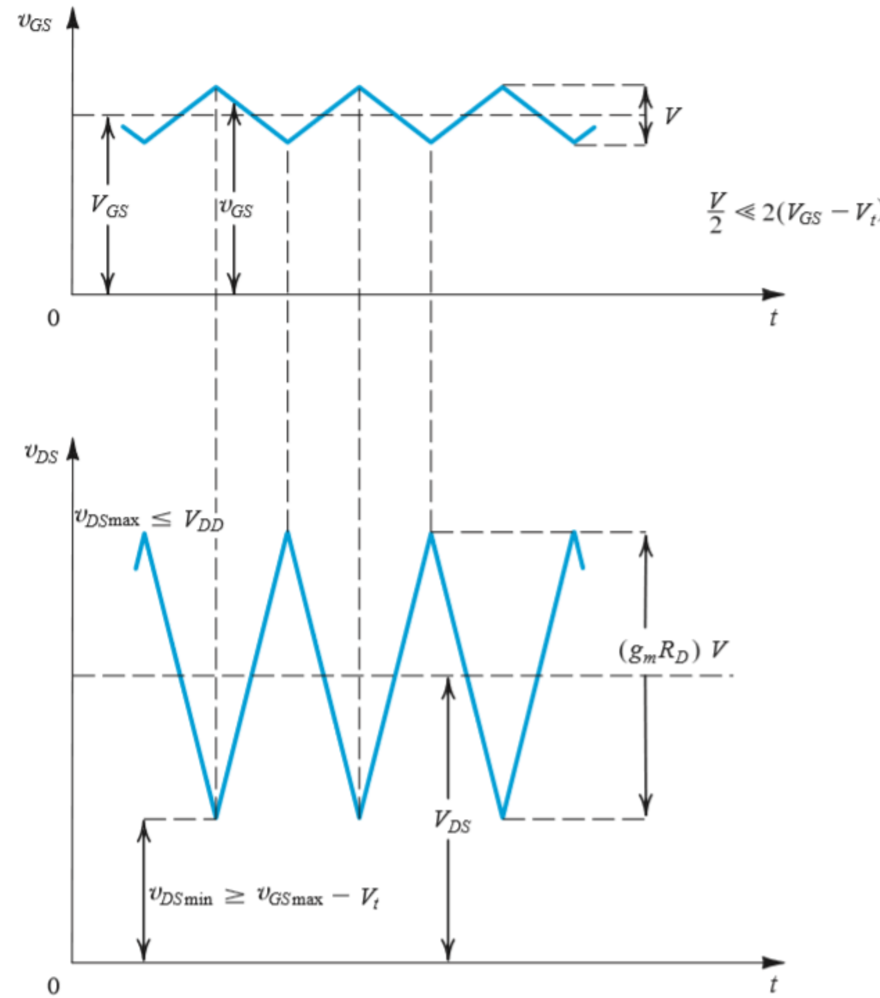


Figure 5.36 Total instantaneous voltages v_{GS} and v_{DS} for the circuit in Fig. 5.34.

Separating the DC Analysis and the Signal Analysis

- Under the small-signal approximation

  → signal quantities are superimposed on dc quantities
- total drain current i_D = equals the dc current I_D plus the signal current i_d
- total drain voltage $v_{DS} = V_{DS} + v_{ds}$ …
- ➔ the analysis and design can be greatly simplified by:
  separating dc (bias calculations) from small-signal calculations

  ➔

1. Establish a stable dc operating point and calculate all dc quantities
2. Perform signal analysis ignoring dc quantities

Steps for Analysis

1. DC Analysis: Assume signal quantities are zeros – Find I_D, V_{GS}, V_{DS} (Capacitors open circuits)
2. Find g_m
3. Signal Analysis: Assume DC quantities are zeros –
   DC Voltage supplies are short circuits –
   DC current sources are open circuits –
   Capacitors are short circuits

3.5 Replace MOSFETs by small signal Model

4. Find i_d and other quantities
5. $i_D = I_D + i_d$

Small-Signal Equivalent-Circuit Models

- From a signal point of view, the FET behaves as a voltage-controlled current source

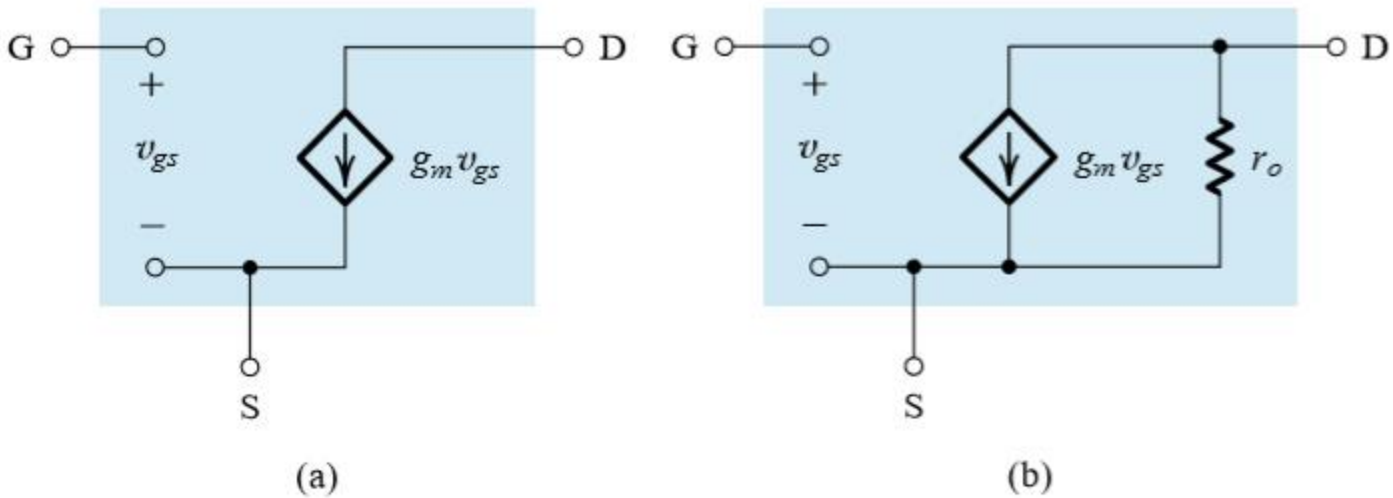


Figure 5.37 Small-signal models for the MOSFET: **(a)** neglecting the dependence of i_D on v_{DS} in saturation (the channel-length modulation effect); and **(b)** including the effect of channel-length modulation, modeled by output resistance $r_o = |V_A|/I_D$.

- where $V_A = 1/\lambda$ is a MOSFET parameter proportional to the channel length
- Voltage gain: $A_v \equiv v_{ds} / v_{gs} = -g_m (R_D || r_o)$
- $g_m = 2I_D/V_{OV}$

Example

Analyze this amplifier circuit to determine:

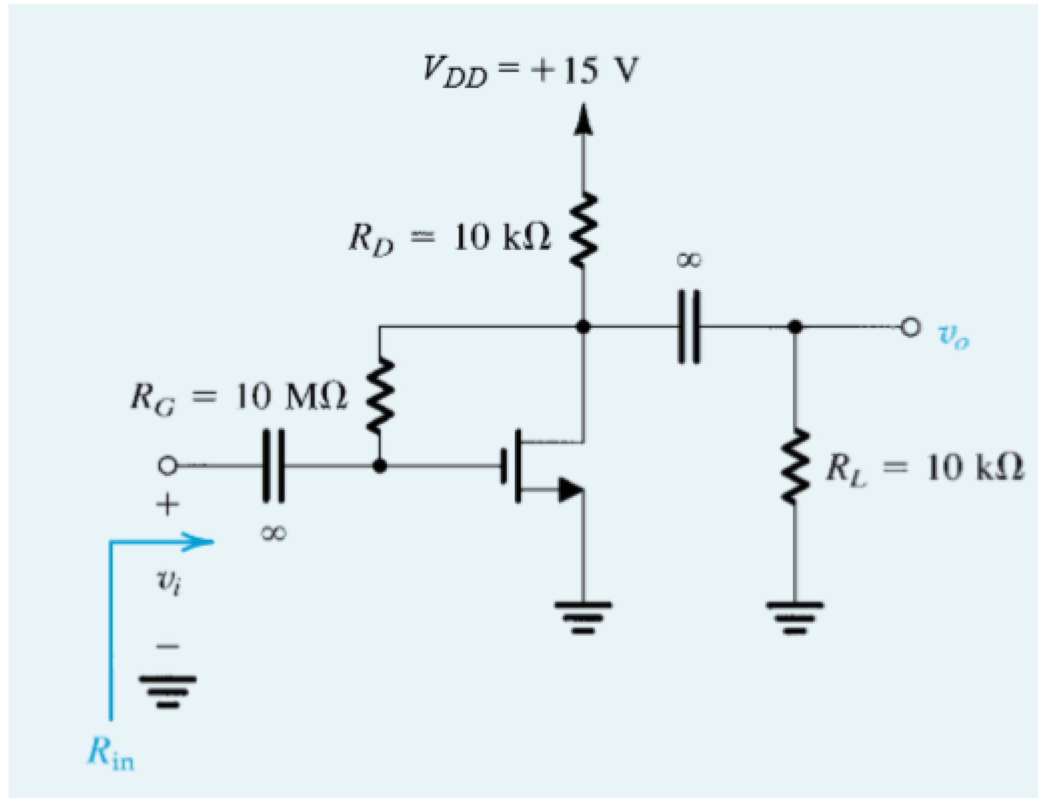
- small-signal voltage gain
- input resistance
- largest allowable input signal

Given:

$V_t = 1.5\,V$; $k'_n(W/L)=0.25\ mA/V_2$;

$V_A = 50\,V$

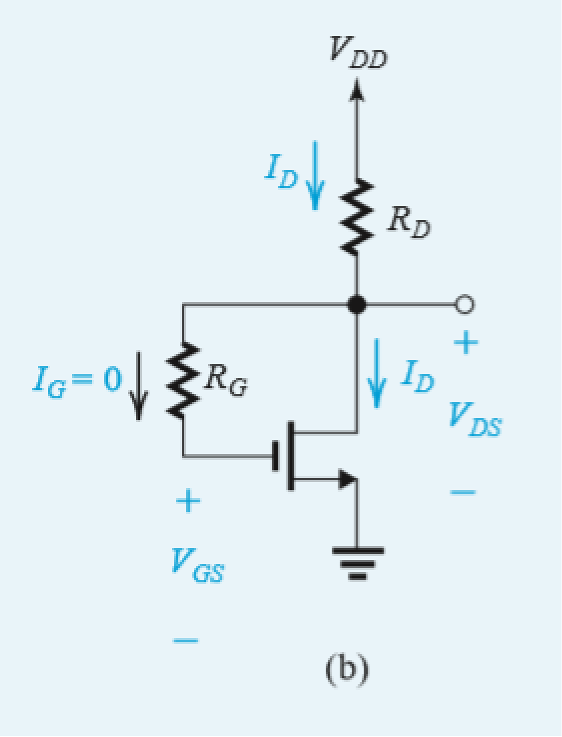
Assume the coupling capacitors to be sufficiently large so as to act as short circuits at the signal frequencies of interest.



Example

Step 1: DC Analysis

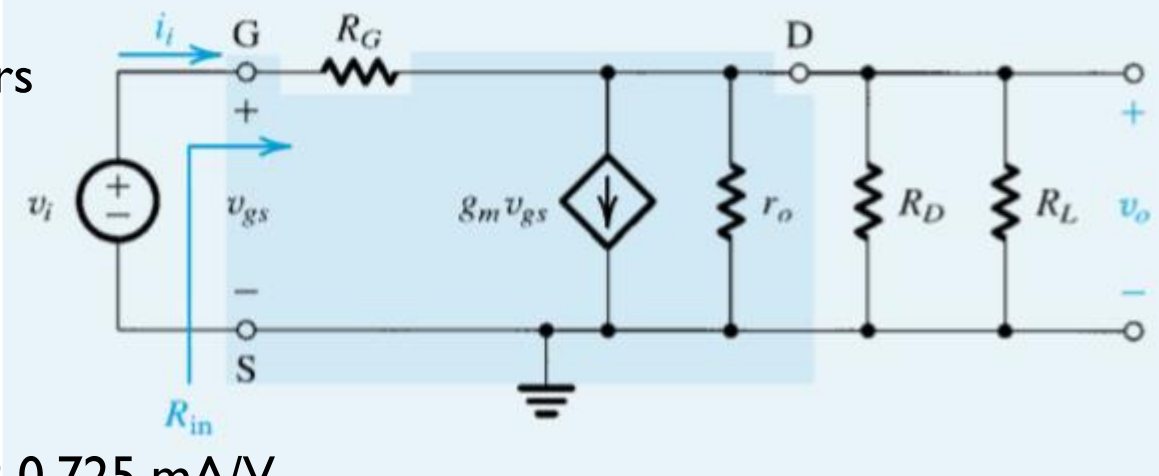
- Determine the dc operating point
- eliminate the input signal v_i,
- Open circuit the two coupling capacitors
- $I_G=0 \rightarrow V_{GS}=V_{DS}=V_{DD}-R_D I_D \rightarrow I_D = (V_{DD}-V_{DS})/R_D$
- SAT: $I_D= \frac{1}{2} * k_n(V_{GS}-V_t)^2 (1+\lambda V_{DS})$
- Solving the two equations:
- I_D=1.06mA and $V_{DS} = V_{GS}$ = 4.4 V
- And V_{OV} = 4.4 - 1.5 = 2.9 V



Example

Step 2: small-signal analysis

- replace coupling capacitors with short circuits
- Replace DC voltage supply with a short circuit to ground.



(c)

- $g_m = k_n V_{OV} = 0.25 \times 2.9 = 0.725$ mA/V
- $r_o = V_A/I_D = 50/1.06 = 47K\Omega$

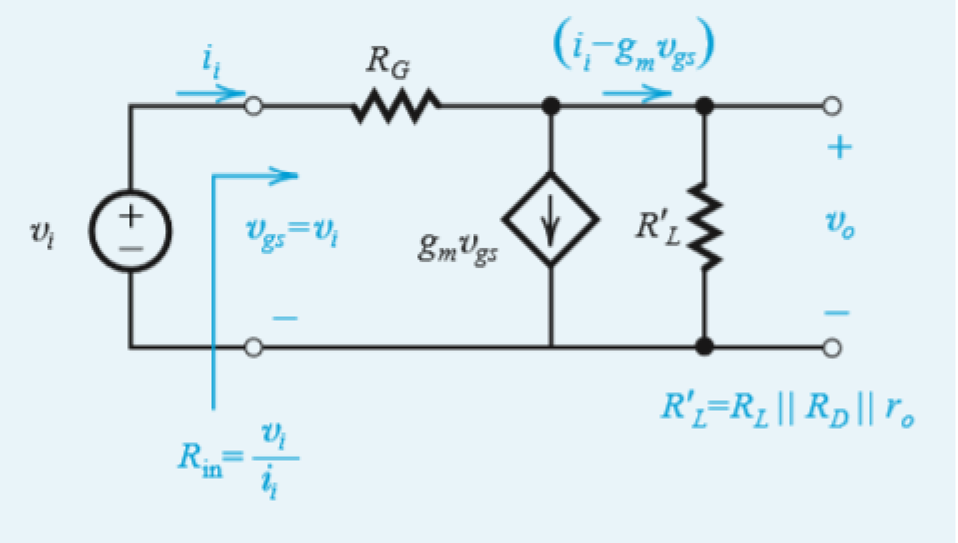
Example

- $R'_L = R_L||R_D||r_o$=10||10||47= 4.52kΩ
- $v_o = (i_i - g_m v_{gs})R'_L$
- $i_i = (v_{gs} - v_o)/R_G$
- $\rightarrow A_v = v_o/v_i =$

$A_v = -g_m R'_L[1-(1/g_m R_G)/(1+(R'_L/R_G)]$

- Since R_G is very large

$A_v \approx -g_m R'_L$ = -3.3 V/V



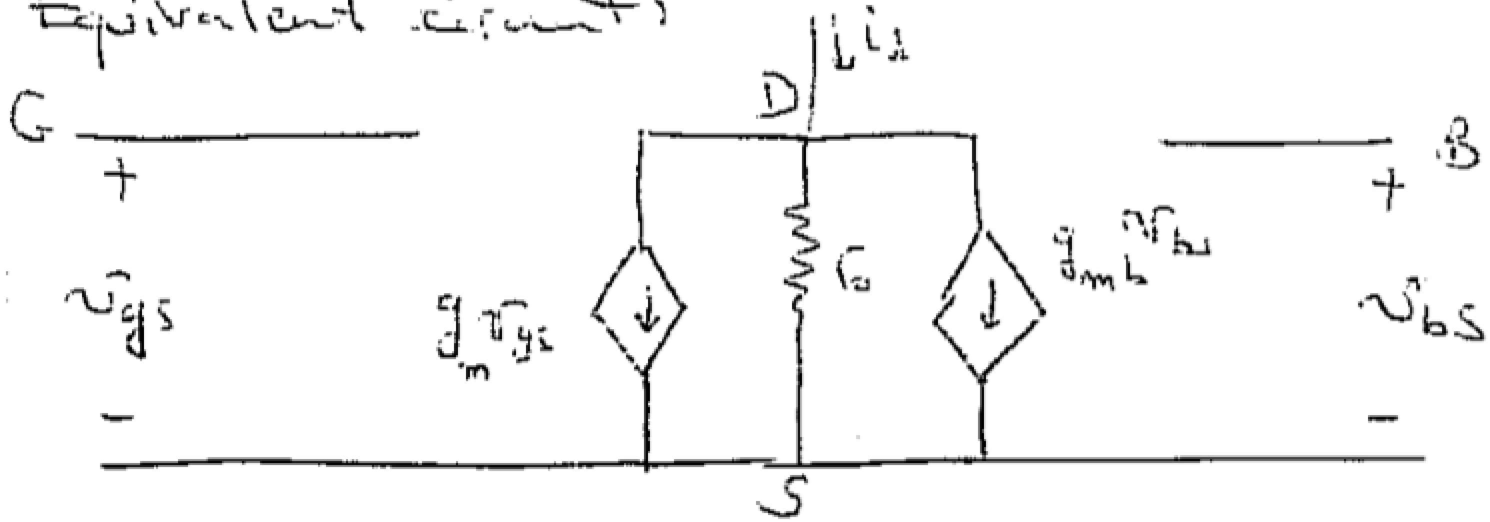
(d)

- $R_{in} = v_i/i_i = v_{gs}/i_i = R_G/(1+g_m R'_L)$ = 2.33MΩ
- Largest allowable input signal is constrained by the need to keep the transistor in saturation at all times → $v_{DS} \geq v_{GS} - V_t$ → $v_{DSmin} = v_{GSmax} - V_t$
- → $V_{DS} - |A_v|\, v_i = V_{GS} + v_i - V_t$ But $V_{DS} = V_{GS}$ → $v_i = V_t/(|A_v|+1)$ → v_i = 1.5/(3.3+1) = 0.35V
- Check $V_{GS} > V_t$ → 4.4 ± 0.35 > 1.5 → we will never reach cutoff region.

Exact Calculations

- Including the Channel Length Modulation and the Body Effect
- $i_D = f(v_{GS}, v_{DS}, v_{BS}) = g_m v_{gs} + g_{ds} v_{ds} + g_{mb} v_{bs}$
- $g_{mb} = X g_m$ where $X = \dfrac{\delta}{2\sqrt{|-2\emptyset_F + V_{SB}|}}$

Equivalent circuit:



The T Equivalent-Circuit Model

All the previously discussed models are referred to as $\pi - Model$

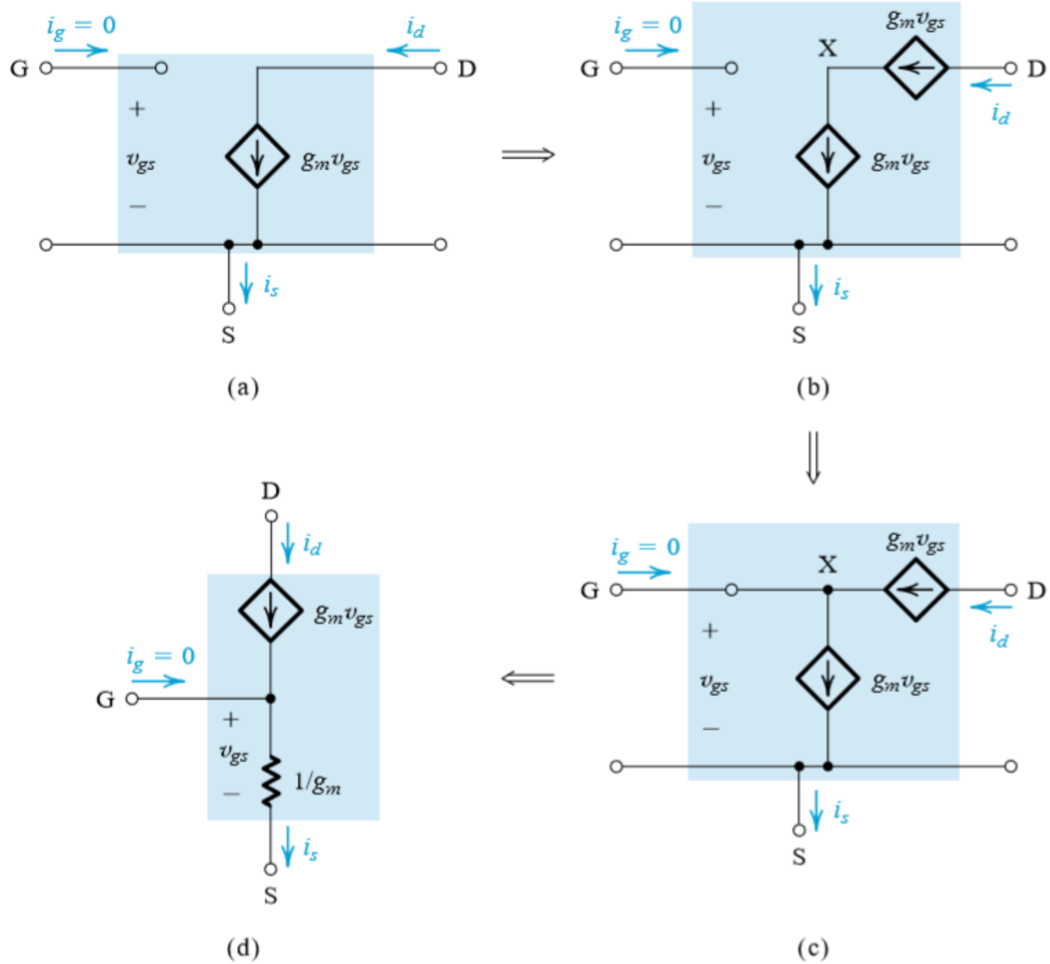


Figure 5.40 Development of the T equivalent-circuit model for the MOSFET. For simplicity, r_o has been omitted; however, it may be added between D and S in the T model of **(d)**.

The T Equivalent-Circuit Model

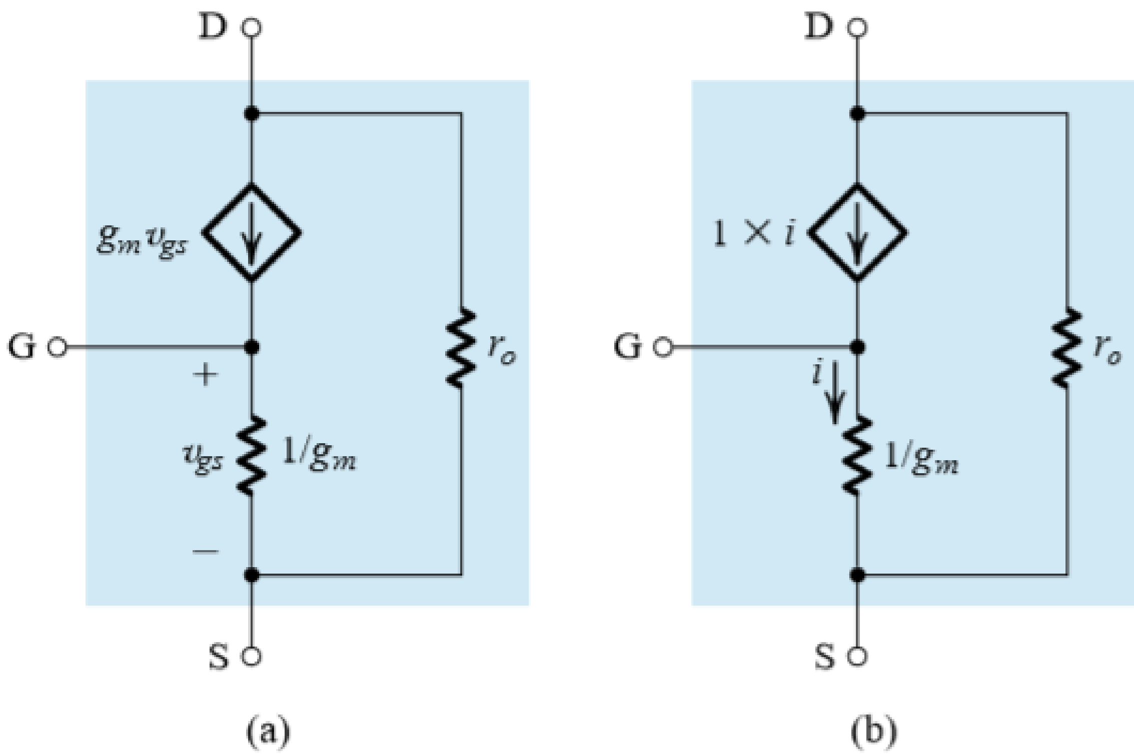


Figure 5.41 **(a)** The T model of the MOSFET augmented with the drain-to-source resistance r_o. **(b)** An alternative representation of the T model.

Small Signal Circuits

Table 5.3 Small-Signal Equivalent-Circuit Models for the MOSFET

Small-Signal Parameters

NMOS transistors

- Transconductance:

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{OV}}$$

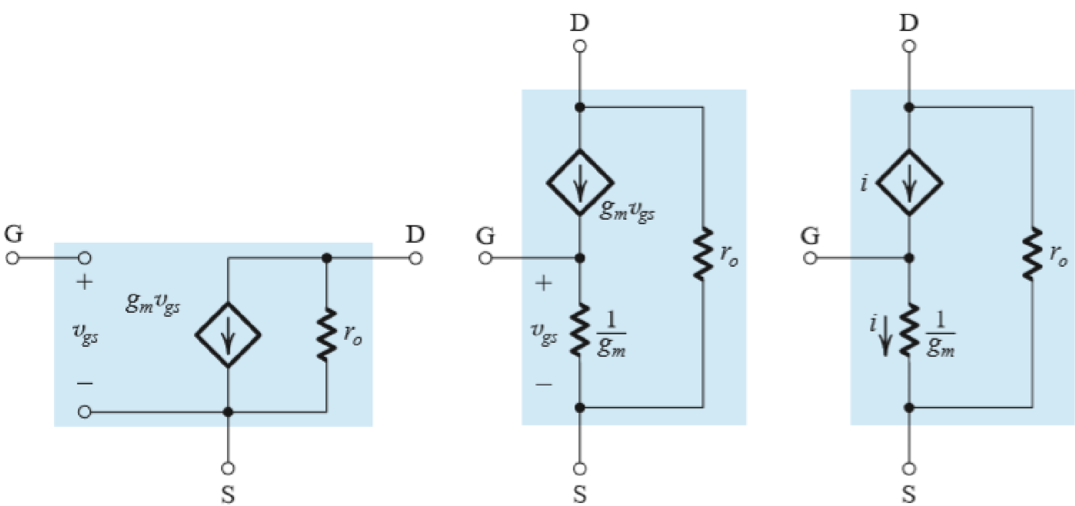
- Output resistance:

$$r_o = V_A / I_D = 1/\lambda I_D$$

PMOS transistors

Same formulas as for NMOS *except* using $|V_{OV}|$, $|V_A|$, and replacing μ_n with μ_p.

Small-Signal Equivalent Circuit Models



Hybrid-π model

T models

Thank you !